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On fluctuating flow of an elastico-viscous fluid past an infinite plate with variable suction

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An exact solution is obtained for the two-dimensional flow of an elastico-viscous (Walters fluid B') incompressible fluid past an infinite porous wall under the following conditions: (i) the suction velocity normal to the plate oscillates in magnitude but not in direction about a non-zero mean; (ii) the free-stream velocity oscillates in time about a constant mean.

The response of the skin-friction to the fluctuating stream and suction velocity is studied for variations in the suction parameter A, the elasticity parameter k and the frequency parameter ω. It is found that the back-flow at the wall is enhanced by k. For the same value of A, the amplitude of the skin-friction decreases with increasing k. Also an increase in k and ω leads to a decrease in the phase of the skin-friction. For moderately large A and k, the phase of the skin-friction may be completely negative.

1. Introduction

Lighthill (1954) initiated an important class of two-dimensional time-dependent flow problems dealing with the response of the boundary layer to external unsteady fluctuations about a mean value. Stuart (1955) examined the interesting features of an oscillating flow over an infinite flat plate with constant suction and no heat transfer between the fluid and the plate. Stuart's problem was later studied by Reddy (1964) under a slip-flow boundary condition. Recently, Messiha (1966) and Soundalgekar (1968) examined Stuart's and Reddy's problems respectively, for the case of variable suction. The hydromagnetic problem corresponding to that of Stuart was investigated by Suryaprakasarao (1962, 1963), and Soundalgekar (1968) generalized Reddy's and Messiha's problems to account for the effects of a magnetic field. All the aspects of velocity field and temperature field in the case of the flow of incompressible, viscous and electrically conducting, or non-conducting, Newtonian fluids were discussed in the above references.

In technological fields, another important class of fluids, called non-Newtonian fluids, are also being studied extensively. The boundary-layer phenomenon in non-Newtonian fluids has also been studied by a number of research workers. One such fluid, whose constitutive equations are characterized by Walters

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(1964, liquid B'), exhibits a boundary-layer phenomenon. The effect of unsteady fluctuations of the free-stream velocity on the flow in the boundary layer of an incompressible elastico-viscous fluid B' past an infinite flat porous plate with constant suction velocity was recently presented in a note by Kaloni (1967). Kaloni has discussed the conditions under which the modifications occur in the flow field of such fluids due to the additional property of elasticity of the fluid. Detailed investigations of the effect of the elastic parameter k of such elastico-viscous fluid have not been carried out by him though it plays an important part in modifying the flow field.

The effect of a variable suction velocity on the flow field of liquid B' has not been attempted so far. Hence the object of the present paper is to study the effect of the variable suction velocity of the form $v_0'(1+\epsilon A\, e^{i\omega' t'})$ as assumed by Messiha with the external flow velocity taken as $U_0'(1+\epsilon\, e^{i\omega' t'})$, following Stuart. It is of interest to study how Stuart's and Messiha's results get modified due to the presence of the elastic property of the fluid. Kaloni has not compared his results with those of Stuart. Hence in the present investigation, the results are compared with those of Stuart's and Messiha's thus bringing out the important contribution to the flow field by the elastic property of the fluid.

In §2, the problem is suitably posed and solved in the case of velocity field. An amplitude and the phase of the skin-friction fluctuations, transient velocity profiles, fluctuating parts of the velocity profiles, are shown on the graphs. A comprehensive summary of results is presented in §3.

A more general form for the free-stream fluctuations will be assumed in a subsequent paper to be presented soon. Such a study will lead to the showing of a number of important aspects of non-Newtonian fluids.

2. Mathematical analysis

The constitutive equations characterizing the elastico-viscous liquid B' are

$$p_{ik} = -p g_{ik} + p'_{ik}, \tag{1}$$

$$p'^{ik}(x,t) = 2\int_{-\infty}^{t} \psi(t-t')\,\frac{\partial x^i}{\partial x'^m}\,\frac{\partial x^k}{\partial x'^r}\,e^{(1)mr}(x',t')\,dt', \tag{2}$$

where p_{ik} is the stress tensor, p an arbitrary isotropic pressure, g_{ik} the metric tensor of a fixed co-ordinate system x^i, x'^i the position at time t' of the element which is instantaneously at the point x^i at time t, $e^{(1)}_{ik}$ the rate of strain tensor and

$$\psi(t-t') = \int_0^\infty \frac{N(\tau)}{\tau}\exp\left[-(t-t')/\tau\right]d\tau,$$

$N(\tau)$ being the distribution function of relaxation times τ. Walters (1962) has shown that in the case of liquids with short memories (i.e. short relaxation times), the equation of state can be written in a simplified form

$$p'^{ik} = 2\eta_0\, e^{(1)ik} - 2k_0\,\frac{\delta}{\delta t}\,e^{(1)ik}, \tag{3}$$

where
$$\eta_0 = \int_0^\infty N(\tau)\,d\tau$$

is the limiting viscosity at small rates of shear,

$$k_0 = \int_0^\infty \tau N(\tau)\, d\tau,$$

and $\delta/\delta t$ denotes the convected differentiation of a tensor quantity, which for any contravariant tensor b^{ik} is given as

$$\frac{\delta b^{ik}}{\delta t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x^m} - b^{im}\frac{\partial v^k}{\partial x^m} - b^{mk}\frac{\partial v^i}{\partial x^m}, \tag{4}$$

where v^i is the velocity vector.

Here the x'-axis is chosen along a two-dimensional infinite plane wall and the y'-axis perpendicular to it. Under these conditions, the flow is independent of x'. Hence, from (1), (3) and (4), the flow of an incompressible elastico-viscous fluid is governed by the following equations of motion and continuity:

$$\rho' \frac{\partial u'}{\partial t'} + \rho v' \frac{\partial u'}{\partial y'} = -\frac{\partial p'}{\partial x'} + \eta_0 \frac{\partial^2 u'}{\partial y'^2} - k_0\left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} + v'\frac{\partial^3 u'}{\partial y'^3} - 3\frac{\partial u'}{\partial y'}\frac{\partial^2 v'}{\partial y'^2} - 2\frac{\partial v'}{\partial y'}\frac{\partial^2 u'}{\partial y'^2}\right), \tag{5}$$

$$\rho' \frac{\partial v'}{\partial t'} + \rho' v' \frac{\partial v'}{\partial y'} = -\frac{\partial p'}{\partial y'} + 2\eta_0 \frac{\partial^2 v'}{\partial y'^2} - 2k_0\left(\frac{\partial^3 v'}{\partial y'^2 \partial t'} + v'\frac{\partial^3 v'}{\partial y'^3} - 3\frac{\partial v'}{\partial y'}\frac{\partial^2 v'}{\partial y'^2}\right), \tag{6}$$

$$\frac{\partial v'}{\partial y'} = 0. \tag{7}$$

It is evident from (7) that v' is a function of time only. Hence, following Messiha, we consider v' as

$$v' = -v_0'(1 + \epsilon A\, e^{i\omega' t'}), \tag{8}$$

where v_0' is a non-zero constant mean suction velocity, ϵ is small and A is a real positive constant such that $\epsilon A \leqslant 1$. The negative sign in (8) indicates that the suction velocity normal to the wall is directed towards the wall.

In view of (7), (5) and (6) reduce to

$$\frac{\partial u'}{\partial t'} - v_0'(1+\epsilon A\, e^{i\omega' t'})\frac{\partial u'}{\partial y'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} - k_0^*\left[\frac{\partial^3 u'}{\partial y'^2 \partial t'} - v_0'(1+\epsilon A\, e^{i\omega' t'})\frac{\partial^3 u'}{\partial y'^3}\right], \tag{9}$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y'}, \tag{10}$$

where $\nu = \eta_0/\rho'$ and $k_0^* = k_0/\rho'$.

Also from (8) and (10), as $\partial p'/\partial y'$ is small in the boundary layer, it can be neglected. Hence the pressure is taken to be constant along any normal and is given by its value outside the boundary layer. If $U'(t')$ is the free-stream velocity, then (9) for the free stream becomes,

$$-\frac{1}{\rho'}\frac{\partial p'}{\partial x'} = \frac{dU'}{dt'}. \tag{11}$$

The equations (9) and (11) give

$$\frac{\partial u'}{\partial t'} - v_0'(1+\epsilon A\, e^{i\omega' t'})\frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + \nu\frac{\partial^2 u'}{\partial y'^2} - k_0^*\left[\frac{\partial^3 u'}{\partial y'^2 \partial t'} - v_0'(1+\epsilon A\, e^{i\omega' t'})\frac{\partial^3 u'}{\partial y'^3}\right]. \tag{12}$$

The boundary conditions are

$$u' = 0 \quad \text{at} \quad y' = 0 \quad \text{and} \quad u' = U'(t') \quad \text{as} \quad y' \to \infty. \tag{13}$$

Let us now consider a periodic free-stream velocity of the form

$$U'(t') = U_0'(1+\epsilon\, e^{i\omega' t'}) \tag{14}$$

and let the velocity in the neighbourhood of the plate be

$$u'(y', t') = U_0'[f_1(y') + \epsilon\, e^{i\omega' t'} f_2(y')], \tag{15}$$

where ω' is the frequency of the fluctuating stream, U_0' = the mean of $U'(t')$, $\epsilon U_0'$ = the amplitude of the free-stream fluctuation, $U_0' f_1$ = the mean velocity in the boundary layer, $\epsilon U_0' f_2$ = the amplitude of the velocity fluctuation in the boundary layer. Substituting (14) and (15) in (12), comparing harmonic terms and neglecting coefficients of ϵ^2, we get

$$kf_1''' + f_1'' + f_1' = 0, \tag{16}$$

$$kf_2''' + f_2''(1 - \tfrac{1}{4}ki\omega) + f_2' - \tfrac{1}{4}i\omega f_2 = -\tfrac{1}{4}i\omega - Af_1' - kAf_1''', \tag{17}$$

where the primes denote differentiation with respect to η and the non-dimensional quantities are defined as follows:

$$\left.\begin{aligned} \eta &= y'v_0'/\nu, \quad t = v_0'^2 t'/4\nu, \quad \omega = 4\nu\omega'/v_0'^2, \\ U &= U'/U_0', \quad u = u'/U_0', \quad k = k_0^* v_0'^2/\nu^2. \end{aligned}\right\} \tag{18}$$

In view of (15), the boundary conditions (13) now reduce to

$$\left.\begin{aligned} f_1 = f_2 = 0 \quad &\text{at} \quad \eta = 0, \\ f_1 = f_2 = 1 \quad &\text{as} \quad \eta \to \infty. \end{aligned}\right\} \tag{19}$$

Equations (16) and (17) are the third-order differential equations when $k \neq 0$, and for $k = 0$ they reduce to equations governing the Newtonian fluid. Hence, the presence of the elasticity of the fluid increases the order of the governing equations from two to three and therefore they need three boundary conditions for their unique solution. But there are prescribed only two boundary conditions in (19). To overcome this difficulty, we follow Beard & Walters (1964) and assume the solution in the form as follows:

$$\left.\begin{aligned} f_1 &= f_{01} + kf_{11} + O(k^2), \\ f_2 &= f_{02} + kf_{12} + O(k^2), \end{aligned}\right\} \tag{20}$$

which is valid for small values of k. As (3) is obtained from (2) on the same reasoning, the approximation is valid.

Substituting (20) in (16) and (17) and equating the coefficients of k we have

$$f''_{01}+f'_{01} = 0, \tag{21}$$

$$f'''_{01}+f''_{11}+f'_{11} = 0, \tag{22}$$

$$f''_{02}+f'_{02}-\tfrac{1}{4}i\omega f_{02} = -\tfrac{1}{4}i\omega - Af'_{01}, \tag{23}$$

$$f''_{12}+f'_{12}-\tfrac{1}{4}i\omega f_{12} = \tfrac{1}{4}i\omega f''_{02}-f'''_{02}-Af'_{11}-Af'''_{01}, \tag{24}$$

with the corresponding boundary conditions

$$\left.\begin{aligned} f_{01} = f_{11} = f_{02} = f_{12} = 0 \quad & \text{at} \quad \eta = 0, \\ f_{01} = f_{02} = 1, \quad f_{11} = f_{12} = 0 \quad & \text{as} \quad \eta \to \infty. \end{aligned}\right\} \tag{25}$$

Solving (21) to (24) under the boundary conditions (25), we have, in view of (20),

$$f_1 = 1 - e^{-\eta} + k\eta\, e^{-\eta}, \tag{26}$$

$$f_2 = 1 - S\,e^{-h\eta} - (1-S)\,e^{-\eta} + k\{(1-S)\,(e^{-h\eta} - (1-\eta)\,e^{-\eta}) + L\eta\, e^{-h\eta}\}, \tag{27}$$

where $S = 1 - 4iA/\omega, \quad h = \tfrac{1}{2}[1+(1+i\omega)^{\frac{1}{2}}]$

and

$$L = \frac{h^2(h+\frac{1}{4}i\omega)\,(1-4iA/\omega)}{(1+i\omega)^{\frac{1}{2}}}. \tag{28}$$

Hence the velocity field in the boundary layer is given by

$$\begin{aligned} u = 1 - e^{-\eta} + k\eta\, e^{-\eta} + \epsilon\, e^{i\omega t}[1 - S\,e^{-h\eta} - (1-S)\,e^{-\eta} \\ + k\{(1-S)\,(e^{-h\eta} - (1-\eta)\,e^{-\eta}) + L\eta\, e^{-h\eta}\}]. \end{aligned} \tag{29}$$

We can now obtain the expression for the shearing stress from (1) and (3) as

$$p'_{x'y'} = \eta_0 \frac{\partial u'}{\partial y'} - k_0 \left(\frac{\partial^2 u'}{\partial y'\,\partial t'} + v' \frac{\partial^2 u'}{\partial y'^2}\right) \tag{30}$$

and in virtue of (18), (30) reduces to

$$p_{xy} = \frac{p'_{x'y'}}{U'_0 v'_0 \rho'} = \frac{\partial u}{\partial \eta} - \tfrac{1}{4}k \left[\frac{\partial^2 u}{\partial \eta\,\partial t} - 4(1+\epsilon A\, e^{i\omega t})\,\frac{\partial^2 u}{\partial \eta^2}\right]. \tag{31}$$

Hence from (29) and (31), we get

$$\begin{aligned} p_{xy}\big|_{\eta=0} = 1 + \epsilon\, e^{i\omega t}[(1-S)\,(1+k-\tfrac{1}{4}i\omega k) + h(S - k(1-S) \\ - \tfrac{1}{4}i\omega k S) + k(L - A - Sh^2)] \end{aligned} \tag{32}$$

on neglecting the coefficient of ϵ^2. Hence from (29),

$$u(y,t) = 1 - e^{-\eta} + k\eta\, e^{-\eta} + \epsilon(M_r \cos \omega t - M_i \sin \omega t), \tag{33}$$

where M_r, M_i are the fluctuating parts of the velocity profile and are given by

$$M_r = 1 + e^{-h_r \eta}\,[(4A/\omega)\,(1+k)\sin h_i \eta - \cos h_i \eta + \eta k\{L_r \cos h_i \eta + L_i \sin h_i \eta\}], \tag{34}$$

$$\begin{aligned} M_i = [\{(4A/\omega)\,(1+k) + \eta k L_i\}\cos h_i \eta + (1 - \eta k L_r)\sin h_i \eta]\, e^{-h_r \eta} \\ - (4A/\omega)\,[1 + k(1-\eta)]\, e^{-\eta}, \end{aligned} \tag{35}$$

where

$$\left.\begin{aligned} h_r &= \tfrac{1}{2} + \tfrac{1}{2}\{\tfrac{1}{2}[(1+\omega^2)^{\frac{1}{2}} + 1]\}^{\frac{1}{2}}, \\ h_i &= \tfrac{1}{2}\{\tfrac{1}{2}[(1+\omega^2)^{\frac{1}{2}} - 1]\}^{\frac{1}{2}}, \end{aligned}\right\} \tag{36}$$

$$\left.\begin{aligned} L_r &= r^{-\frac{1}{2}}\left[\frac{r^{\frac{1}{2}}}{2}+\frac{r^{\frac{1}{2}}}{2}\left(\frac{r+1}{2}\right)^{\frac{1}{2}}-\frac{r^2-1}{16}\left(\frac{r+1}{2r}\right)^{\frac{1}{2}}\right. \\ &\qquad \left.+\frac{4A}{\omega}\left\{\frac{\omega r^{\frac{1}{2}}}{4}+\frac{r^{\frac{1}{2}}}{2}\left(\frac{r-1}{2}\right)^{\frac{1}{2}}+\frac{\omega^2}{16}\left(\frac{r-1}{2r}\right)^{\frac{1}{2}}\right\}\right], \\ L_i &= r^{-\frac{1}{2}}\left[\frac{\omega r^{\frac{1}{2}}}{4}+\frac{r^{\frac{1}{2}}}{2}\left(\frac{r-1}{2}\right)^{\frac{1}{2}}+\frac{r^2-1}{16}\left(\frac{r-1}{2r}\right)^{\frac{1}{2}}\right. \\ &\qquad \left.-\frac{4A}{\omega}\left\{\frac{r^{\frac{1}{2}}}{2}+\frac{r^{\frac{1}{2}}}{2}\left(\frac{r+1}{2}\right)^{\frac{1}{2}}-\frac{r^2-1}{16}\left(\frac{r+1}{2r}\right)^{\frac{1}{2}}\right\}\right], \end{aligned}\right\} \tag{37}$$

$$r^2 = 1+\omega^2.$$

Also from (32), we have

$$p_{xy} = 1+\epsilon|B|\cos(\omega t+\alpha), \tag{38}$$

where
$$B = (1-S)(1+k-\tfrac{1}{4}i\omega k)+h(S-k(1-S)-\tfrac{1}{4}i\omega kS)+k(L-A-Sh^2) \tag{39}$$
$$= B_r+iB_i,$$
$$\alpha = \tan^{-1}(B_i/B_r), \tag{40}$$

$$\left.\begin{aligned} B_r &= (1-kA)h_r+\left(\frac{4A}{\omega}(1+k)+\frac{k\omega}{4}\right)h_i+kL_r-k(h_r^2-h_i^2)-\frac{8Akh_rh_i}{\omega}, \\ B_i &= \frac{4A}{\omega}(1+k)-h_r\left(\frac{4A}{\omega}(1+k)+\frac{k\omega}{4}\right)+h_i(1-kA)+kL_i+\frac{4Ak}{\omega}(h_r^2-h_i^2)-2kh_rh_i. \end{aligned}\right\} \tag{41}$$

For small values of the frequency parameter ω, we have

$$\begin{aligned} h_r &= 1+\tfrac{1}{16}\omega^2+O(\omega^4), \\ h_i &= \tfrac{1}{4}(\omega-\tfrac{1}{8}\omega^3)+O(\omega^5), \\ L_r &= \tfrac{1}{2}[1+2A-(1+\tfrac{1}{16}A)\tfrac{1}{2}\omega^2]+O(\omega^4), \\ L_i &= (1/16\omega)[(4+3A)\omega^2-48A]+O(\omega^3), \\ M_r &= 1+e^{-\eta}[-1+(A+2Ak+\tfrac{1}{2}k)\eta-\tfrac{3}{4}Ak\eta^2+\omega^2\{-\tfrac{1}{8}Ak \\ &\qquad +\tfrac{1}{64}(4-8A-16k-Ak)\eta+\tfrac{1}{192}(15-12A-6k-17Ak)\eta^2 \\ &\qquad +\tfrac{1}{192}(9-A)\eta^3+(Ak/128)\eta^4\}]+O(\omega^3), \\ M_i &= e^{-\eta}\{-(7/\omega)Ak\eta+\tfrac{1}{4}\omega\eta[1-A-\tfrac{1}{2}A\eta+k(1-\tfrac{1}{4}A-\tfrac{1}{2}\eta-\tfrac{3}{4}A\eta \\ &\qquad +\tfrac{1}{8}3A\eta^2)]\}+O(\omega^3) \end{aligned} \tag{42}$$

and for large values of ω,

$$\begin{aligned} h_r &= \frac{1}{2}\left[1+(\tfrac{1}{2}\omega)^{\frac{1}{2}}+\frac{1}{2(2\omega)^{\frac{1}{2}}}+\frac{1}{8(2)^{\frac{1}{2}}\omega^{\frac{3}{2}}}\right], \\ h_i &= \frac{1}{2(2)^{\frac{1}{2}}}\left(\omega^{\frac{1}{2}}-\frac{1}{2(\omega)^{\frac{1}{2}}}+\frac{1}{8\omega^{\frac{3}{2}}}\right), \\ L_r &= -\frac{1}{16(2)^{\frac{1}{2}}}\omega^{\frac{3}{2}}+\frac{15+8A}{32(2)^{\frac{1}{2}}}\omega^{\frac{1}{2}}+(\tfrac{1}{2}+A) \\ &\qquad +\frac{1}{(2\omega)^{\frac{1}{2}}}\left(\frac{33+60A}{128}\right)+\frac{A}{16\omega}+\frac{7-33A}{32(8\omega)^{\frac{3}{2}}}, \end{aligned}$$

$$L_i = \frac{1}{\omega^{\frac{1}{2}}}\left[\frac{\omega^{\frac{3}{2}}}{4} + \frac{1}{16(2)^{\frac{1}{2}}} + \frac{\omega}{2(2)^{\frac{1}{2}}}\left(\frac{15}{16} - \frac{33}{64\omega} + \frac{\omega}{8} + \frac{51}{128\omega^2}\right)\right]$$
$$- \frac{4A}{\omega^{\frac{3}{2}}}\left[-\frac{\omega^2}{16(2)^{\frac{1}{2}}} + \frac{15\omega}{32(2)^{\frac{1}{2}}} + \frac{\omega^{\frac{1}{2}}}{2} + \frac{33}{128(2)^{\frac{1}{2}}} + \frac{7}{32(2)^{\frac{1}{2}}\omega}\right],$$

$$M = \left(\frac{4iA}{\omega} - 1\right)\exp\left[-\tfrac{1}{2}\eta(i\omega)^{\frac{1}{2}}\right] + 1 - \frac{4iA}{\omega}e^{-\eta}$$
$$+ k\left[\frac{4iA}{\omega}\exp\left[-\tfrac{1}{2}\eta(i\omega)^{\frac{1}{2}}\right] + L\eta\exp\left[-\tfrac{1}{2}\eta(i\omega)^{\frac{1}{2}}\right] - \frac{4iA}{\omega}(1+\eta)e^{-\eta}\right]. \quad (43)$$

From (33), we get for $\omega t = \frac{1}{2}\pi$,

$$u = 1 - e^{-\eta} + k\eta\, e^{-\eta} - \epsilon M_i, \quad (44)$$

where M_i is given by (35).

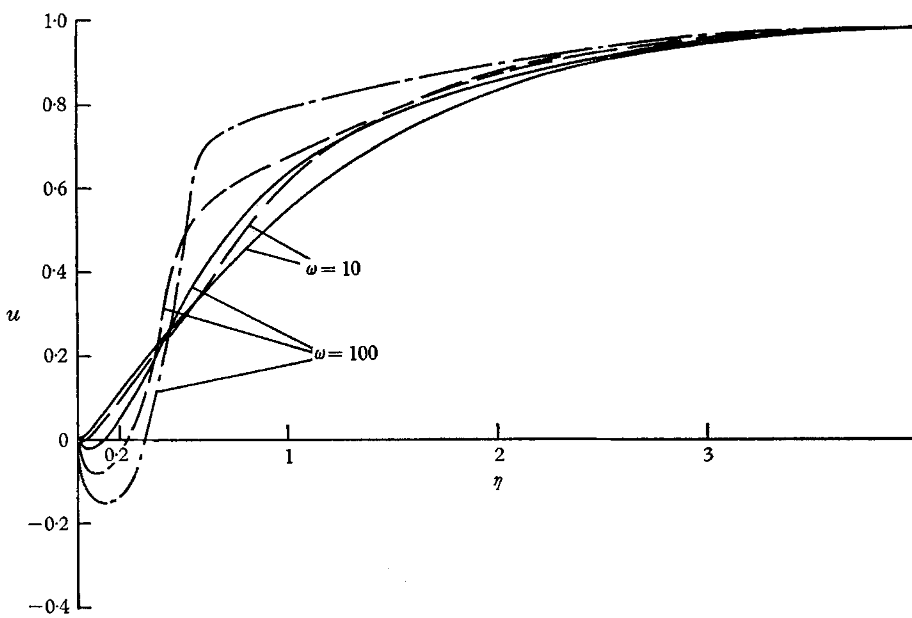

FIGURE 1. Velocity profiles. $\omega t = \frac{1}{2}\pi$, $\epsilon = 0{\cdot}5$, $\omega = 10, 100$, $A = 0$. ———, $k = 0$; — — —, $k = 0{\cdot}05$; —·—·—, $k = 0{\cdot}1$.

In order to study the effect of the elastic property of the elastico-viscous fluid on the distribution of the velocity profiles near the wall, both in the case of constant and variable suction, we have plotted u against η in figures 1 to 4 for different values of A, ω, k and ϵ. It was observed by Stuart that for $\epsilon = 0{\cdot}5$ and $\omega = 100$ ($\frac{1}{4}\omega = \lambda$ in Stuart's case) the velocity is negative near the wall, which is also shown in our figure 1 for $k = 0$. But the graphs in figure 1 for non-zero values of k are particularly interesting in the sense that, with an increase in the value of k, the velocity becomes still more negative near the wall for $\epsilon = 0{\cdot}5$ and $\omega = 100$. This leads us to study the nature of the velocity profiles for smaller values of ϵ and ω. Thus, it can be seen from the velocity profiles in figure 2 that in the case of elastico-viscous fluids, for very small values of k, the velocity is observed to be negative even for smaller values of ϵ and ω viz. $\epsilon = 0{\cdot}2$ and $\omega = 80$. Hence, in the

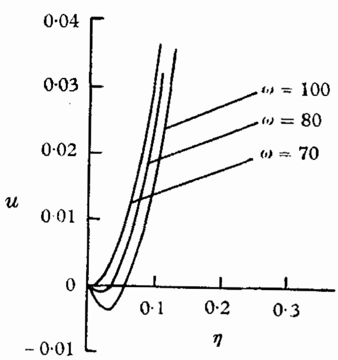

FIGURE 2. Velocity profiles. $\omega t = \frac{1}{2}\pi$, $k = 0{\cdot}05$, $A = 0$, $\epsilon = 0{\cdot}2$.

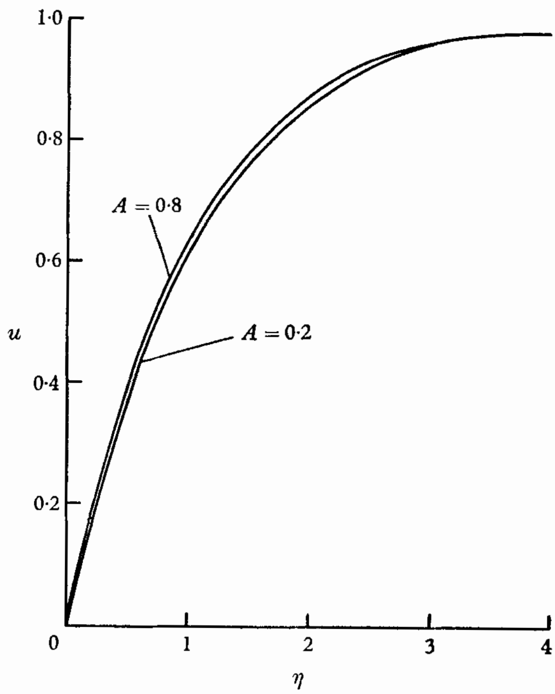

FIGURE 3. Velocity profiles. $\epsilon = 0{\cdot}2$, $\omega t = \frac{1}{2}\pi$, $\omega = 10$, $k = 0{\cdot}05$.

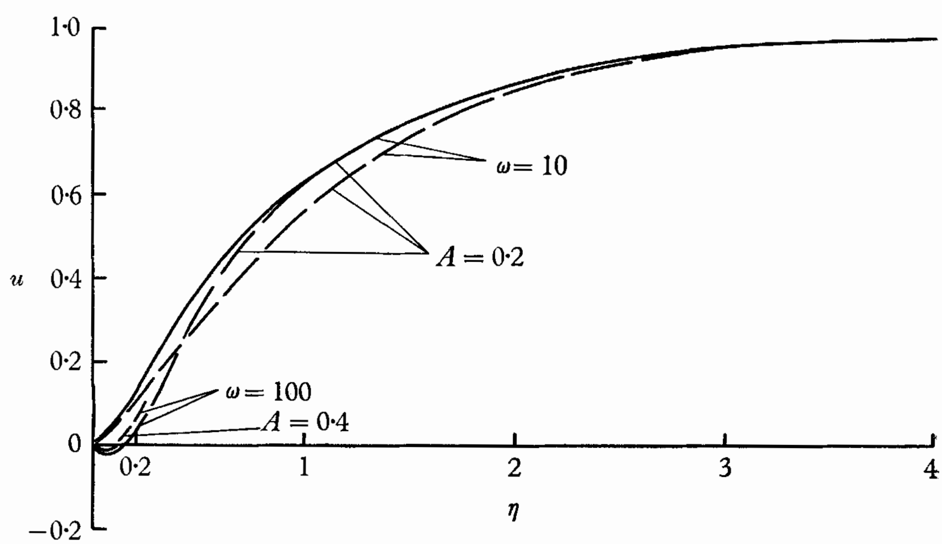

FIGURE 4. Velocity profiles. $\omega t = \frac{1}{2}\pi$, $K = 0$, $A = 0{\cdot}2$, $0{\cdot}4$. ———, $\epsilon = 0{\cdot}2$; — — —, $\epsilon = 0{\cdot}5$.

case of constant suction velocity, the separation occurs at the wall even for small values of ϵ and ω. Figure 4 is prepared to bring out the effects of the variable suction velocity on the separation of the fluid at the wall. This is Messiha's case. Messiha has not discussed the nature of the velocity profiles at large ω, in the presence of the variable suction velocity. From figure 4, it is evident that at large ω, the velocity approaches to a positive value near the wall when there is an increase in A. Hence to avoid the separation near the wall, variable suction may be employed.

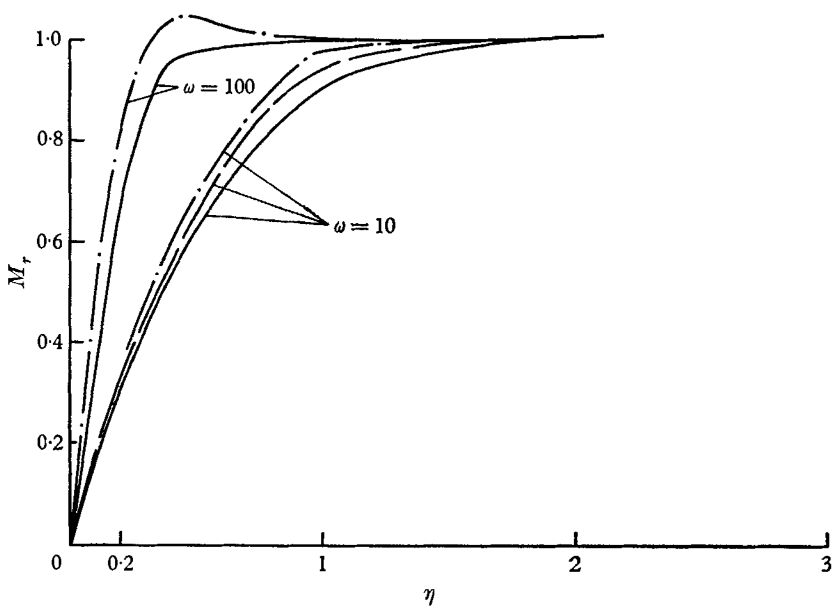

FIGURE 5. Fluctuating part of the velocity profile. $A = 0$, $\omega = 10$, 100. ———, $k = 0$; — — — —, $k = 0{\cdot}05$; —·—·—, $k = 0{\cdot}1$.

In figures 5–8, the details about the fluctuating parts are shown for comparison purposes. In case of constant suction velocity ($A = 0$), an increase in k or ω leads to an increase in M_r and M_i. Figure 7 is particularly interesting because it illustrates the effects of k at large ω on M_i when suction velocity is constant. In the case of the fluids with elastic property, at large ω, there is a sudden rise and fall of M_i near the wall, which is not observed in ordinary Newtonian fluids. Also, from figures 6 and 7, one can conclude that an increase in A or ω leads to an increase in M_r but a decrease in M_i.

Figure 9 illustrates the effects of A and k on the amplitude of the skin-friction. Messiha observed that an increase in A leads to an increase in the amplitude of the skin-friction. The same is also true in case of elastico-viscous fluids (type B'). $|B|$ increases with increasing ω and A. But for the same value of A, an increase in k leads to a decrease in $|B|$.

Figures 10 and 11 illustrate the effects of k and A on the skin-friction phase. It was observed by Stuart that the skin-friction phase rises from zero at zero frequency, to $\frac{1}{4}\pi$ at very high frequencies. This is shown in figure 10, where $k = 0$,

$A = 0$ corresponds to Stuart's case. The other three curves show the effect of k on the phase of the skin-friction. It is interesting to note that an increase in k leads to a decrease in the phase of the skin-friction at large ω. As a particular case, one can observe from figure 10 that $\tan\alpha = 0$ when $\omega = 57$ and $k = 0{\cdot}1$, from which we can conclude that the skin-friction oscillates in phase with the on-coming fluctuating main-stream. For $\omega > 57$, the phase of the skin-friction is negative.

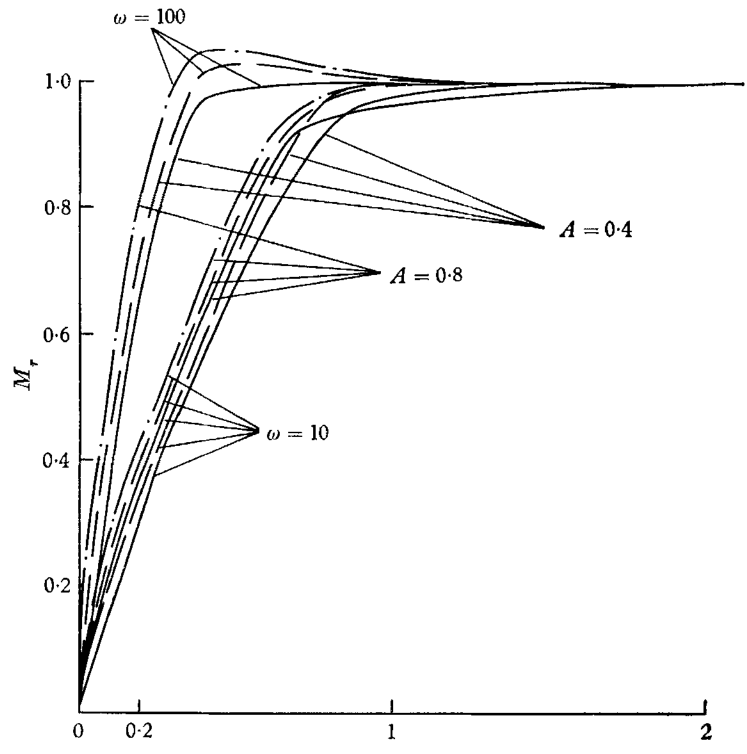

FIGURE 6. Fluctuating parts of velocity profiles. ———, $k = 0$; — — —, $k = 0{\cdot}05$; —·—·—, $k = 0{\cdot}1$.

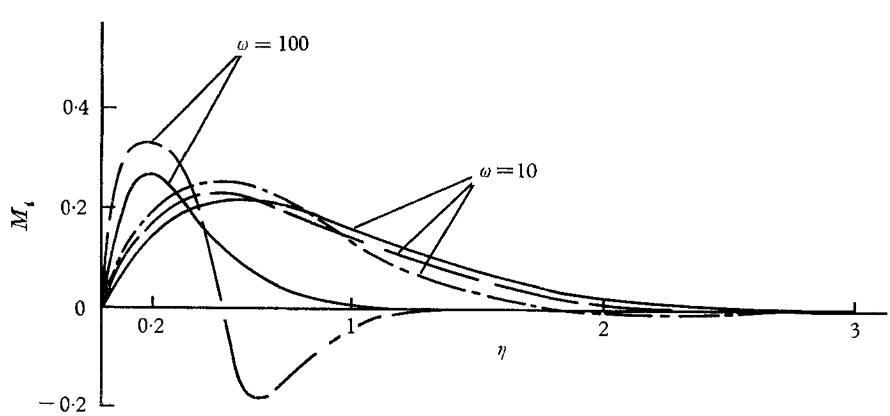

FIGURE 7. Fluctuating part of velocity profile. $A = 0$. ———, $k = 0$; — — —, $k = 0{\cdot}05$; —·—·—, $k = 0{\cdot}1$.

In figure 11, the results are compared with those of Messiha who observed that an increase in A leads to a decrease in the phase of the skin-friction but it increases with ω. This is shown in figure 11 by continuous lines. Messiha has also commented that for small values of the frequency parameter ω, the phase of the skin-friction may be negative. The same is true for elastico-viscous fluids (liquid B'). An increase in k leads to a decrease in the phase as in the case of constant suction velocity. At large ω, the trend is again towards a decrease.

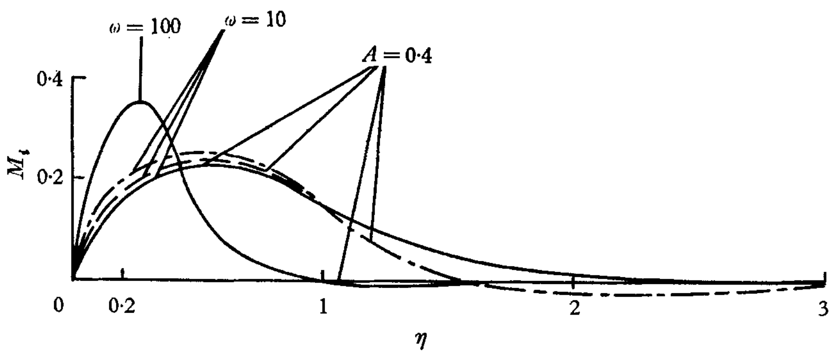

FIGURE 8. Fluctuating part of velocity profile. $A = 0{\cdot}4,\ 0{\cdot}8,\ \omega = 10,\ 100$. ———, $k = 0$; — — —, $k = 0{\cdot}05$; —·—·—, $k = 0{\cdot}1$.

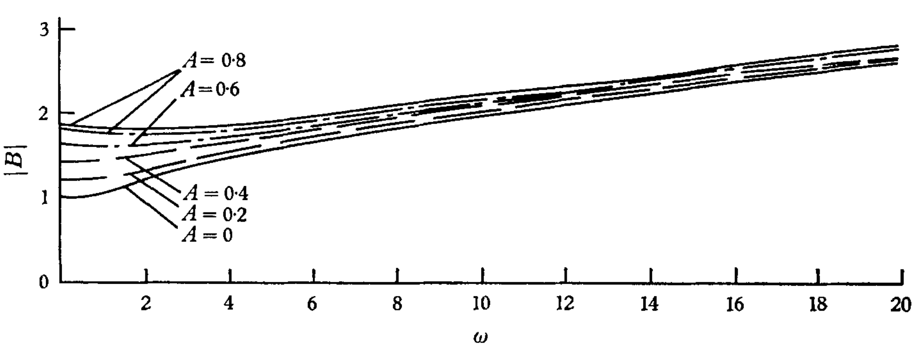

FIGURE 9. Amplitude of the skin-friction against frequency ω. $A = 0,\ 0{\cdot}2,\ 0{\cdot}4,\ 0{\cdot}6,\ 0{\cdot}8$. ———, $k = 0$; — — —, $k = 0{\cdot}05$; —·—·—, $k = 0{\cdot}1$.

3. Conclusions

We summarize here some of our important results. (*a*) Due to the elastic property of the fluid, the back-flow occurs at the wall at values of ϵ and ω smaller than those in case of Newtonian fluids. (*b*) The fluctuating parts M_r and M_i increase with increasing k, the elastic parameter. (*c*) In the case of constant suction velocity ($A = 0$), the amplitude of the skin-friction is affected significantly at large values of ω. An increase in A and k leads to an increase in the amplitude of the skin-friction. But for the same A, an increase in k leads to a decrease in $|B|$. (*d*) In the case of constant suction velocity, an increase in k and ω leads to a decrease in the phase of the skin-friction. For moderately large k, the phase reduces to zero and then becomes negative even for lesser values of ω. The phase of the skin-friction decreases with increasing A. For moderately large A and k, the phase may become negative for all values of ω.

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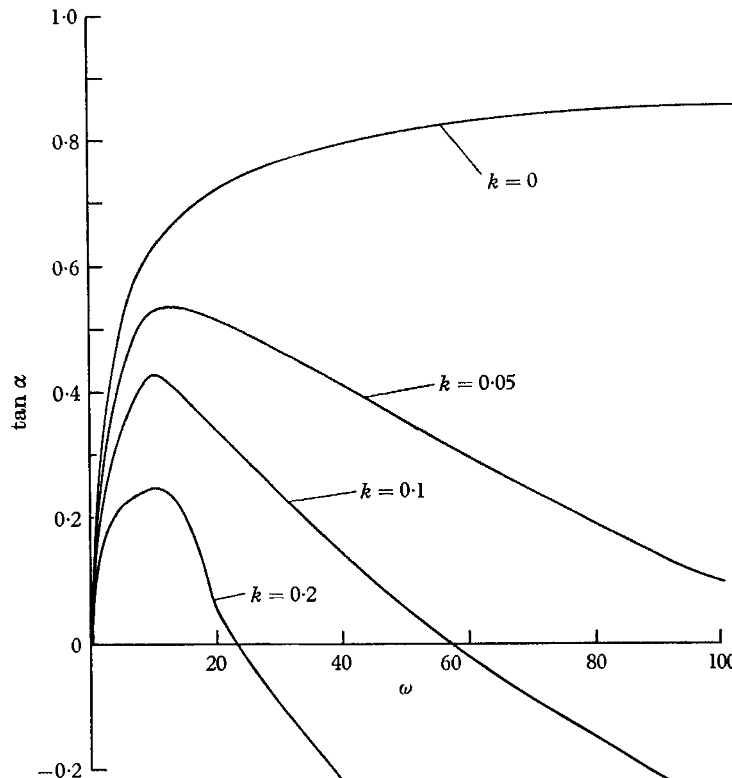

FIGURE 10. Skin-friction phase *vs*. ω. $A = 0$.

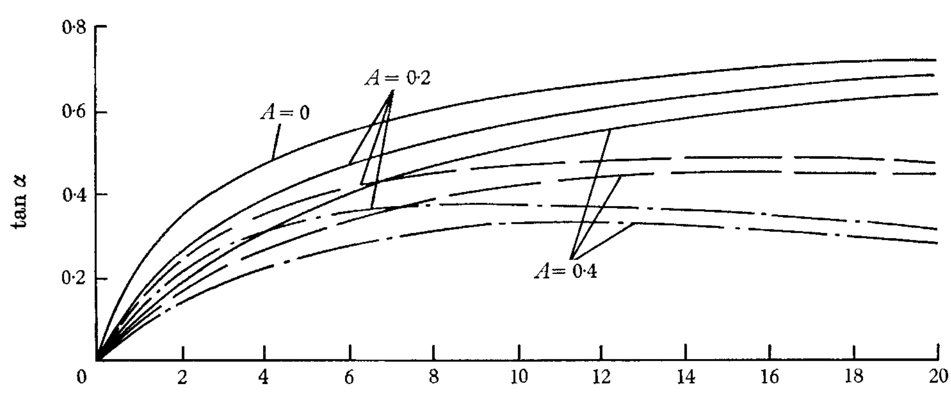

FIGURE 11. Skin-friction phase *vs*. ω. $A = 0$, 0·2, 0·4. ———, $k = 0$; — — —, $k = 0·05$; —·—·—, $k = 0·1$.

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